Lecture 14 Summary

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Tunneling in to Superconductors

Tunneling into and out of a superconductor gives insights in to the density of states as well as the pairing mechanism.

0.1 Tunneling Hamiltonian

We imagine two metals separated by an insulating tunnel barrier. A potential difference is applied between the two metals and the resulting net current is measured. In other words, one measures the "I-V Curve" of the junction. Here we consider only single particle (as opposed to Cooper pair) tunneling. The tunneling Hamiltonian is

$H_T = \sum_{\sigma,k,q} T_{kq} c^+_{k\sigma} c_{q\sigma} + \sum_{\sigma,k,q} T^*_{qk} c^+_{q\sigma} c_{k\sigma}$,

where the first term is for forward tunneling and the second term is for reverse tunneling. Momenta k, q refer to the left and right metal, respectively. This assumes no spin-flip in the tunneling process.

Josephson introduced new operators to create electrons and holes in the metals with probability unity. Because the insulating barrier does not support quasi-particle states, to describe tunneling one has to extract an electron or hole from one metal, realizing the particle in some sense, and then deposit it in the other metal. This essentially destroys the coherence effects discussed in the last lecture, which arise from the fact that Bogoliubon's are a coherent superposition of electron and hole.

Josephson's electron and hole creation operators are,

$\gamma^+_{ek0} = u^*_k c_{k,\uparrow} - v^*_k S^+_k c_{-k\downarrow}$

$\gamma^+_{hk0} = u^*_k S_k c^*_{k,\uparrow} - v^*_k c_{-k\downarrow}$,

where S^+_k creates a $(k,\uparrow)$, $(-k,\downarrow)$ Cooper pair.

In fact one can show that $\gamma^+_{hk0} = S_k \gamma^+_{ek0}$. These excitations still have energy $E_k = \sqrt{\xi^2_k + \Delta^2_k}$.

0.2 Tunnel Current

Because coherence effects are lost, one can use a semiconductor model for tunneling. The superconducting density of states is reflected about the chemical potential and all states below the Fermi energy are filled at zero temperature, and all states above are empty. At finite temperature the Fermi distribution $f(E)$ describes the number of quasiparticles in the excited states.

The net tunneling current is given by,
$I = A\int_{-\infty}^{+\infty} |T|^2 N_L(E)N_R(E+eV)\,[f(E)-f(E+eV)]\,dE$, where L refers to the left metal and R refers to the right metal, and V is the potential difference. Note that the energy integrals are interrupted in the range where the densities of states are zero.
The tunneling current is clearly dominated by the energy dependence of the densities of states $N(E)$ of the two banks.

There are three cases to consider.

0.3 N-I-N, N-I-S and S-I-S Tunneling

In the N-I-N case we take the DOS to be constant near the Fermi energy, and the I-V curve is linear in voltage: $I_{NIN} = G_{NN}V$ with $G_{NN} = A\,|T|^2\,N_L(0)N_R(0)e$.

In the S-I-N case at zero temperature there will be no current until the Fermi energy of the normal metal lines up with the gap edge in the superconductor, i.e. $eV = \pm\Delta$, at which point the current quickly rises and then eventually increases linearly with voltage. At finite temperature the turn-on at Δ is washed out by the excited quasiparticles in the normal metal.
The differential conductance can be written as,
$G_{NS} = \frac{dI_{NS}}{dV} = G_{NN}\int_{-\infty}^{+\infty} \frac{N_S(E)}{N_N(E)}\left[-\frac{\partial f(E+eV)}{\partial(eV)}\right]dE.$
The $N_S(E)$ DOS term is strongly peaked at Δ and the Fermi function derivative becomes more and more strongly peaked as temperature approaches zero. In the limit of zero temperature it becomes a delta function and picks out the superconducting DOS at the voltage bias. As such the low-temperature differential conductance directly measure the superconducting DOS.

In the S-I-S case there will be current peaks at voltages that line up the gap edges for both $eV = \Delta_L + \Delta_R$ and for $eV = |\Delta_L - \Delta_R|$.